Closing Thurs: HW 10.2

Closing Tues: HW 10.3, 11.1&2(part 1)

10.3 Curve Sketching Summary
Local and Global Max/Min

Entry Task: Consider

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$

1. Find all critical values.
2. Plug the critical values into the 2^{nd} derivative. What can you conclude?
3. Draw the 1^{st} and 2^{nd} deriv. analysis number lines.

Summary

For any question about "**increasing, decreasing, local max/min**", identify the function in question, $y = f(x)$, then:

Step 1: Solve $f'(x) = 0$

Step 2:

(option 1): 1st Deriv. Test

Draw 1st deriv. analysis number line.

Make appropriate conclusions.

(option 2): 2nd Deriv. Test

Plug critical numbers into 2nd deriv.

- $f'(a) = 0, f''(a) > 0$ ⇒local min
- $f'(a) = 0, f''(a) < 0$ ⇒local max

For any question about "**concave up/down or inflection points**", identify the function in question, $y = f(x)$, then:

Step 1: Solve $f''(x) = 0$

Step 2:

Draw 2st deriv. analysis number line.

Make appropriate conclusions.

Global Max/Min:

Given $y = f(x)$ and an interval $a \leq x \leq b$

The ***global maximum*** (or *absolute max*) of $f(x)$ on the interval is the highest overall y-value on that interval.

The ***global minimum*** (or *absolute min*) of $f(x)$ on the interval is the highest overall y-value on that interval

Key (awesome) Observation
(Extreme Value Thm)
The global max/min can only occur at:

a. critical values

   OR

b. endpoints.

For any question about "**global max/min**", identify the function and interval in question, $y = f(x)$ and $a \leq x \leq b$ then:

Step 1: Solve $f'(x) = 0$

Step 2:

a. Plug the critical values into the original function.
b. Plug the endpoints into the original function.

At the end of step 2:
The biggest output is the global max.
The smallest output is the global min.

Example (same function from entry task):

On the interval $-2 \leq x \leq 10$, find the global max and min of

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$

Example (from HW 10.3/10):

The total revenue (in thousand dollars) for selling q thousand Framits is given by

$$TR(q) = \frac{1}{6}q^4 - \frac{31}{6}q^3 + 55q^2 + 200q$$

Part (c) Find the global max and global min of marginal revenue over the interval q = 0 to q = 12.

Example: (like the last problem in HW)

Given $g(x) = \frac{1}{4}x^2 - 4x + 25$ and

$$S(x) = \frac{g(x)}{x}$$

If x is between 1 and 20, what is the smallest possible value of S(x)?

Example: (like problems 5-9 of HW)

Given the monthly average cost and price for producing and selling q items:

$$AC(q) = \frac{36000}{q} + 100 + q$$

$$p = 1700$$

If production is limited to 400 items per month, what quantity maximizes profit?